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**R/P094**

# the use of amplifiers with photomultipliers

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A G Wright Electron Tubes Ltd,
Bury Street, Ruislip HA4 7TA UK

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## 1 introduction

Despite advances in avalanche photodiodes, CCDs and CMOS detectors, the vacuum photomultiplier (pmt) is still the most sensitive and versatile of light detectors. In very low light level applications, the photomultiplier and certain GEM devices are the only large area (>0.1 $cm^2$) detectors capable of detecting events containing a single photon. The pmt offers a unique combination of large area light transducer together with an amplifier of outstanding performance (known as an electron multiplier). Electron multipliers have the following exceptional attributes:

- high gain - up to $10^9$
- wide bandwidth – up to 1 GHz
- exceptionally low noise – excess noise with Fano* factor < 1.2
- zero offset
- low and constant output capacitance

*A noiseless amplifier has a Fano factor of unity whereas an avalanche photodiode has a noise factor of between 2 and 4.

Given such performance, why should there ever be need for additional amplification?

It is important to appreciate that amplifiers perform operations other than amplification and frequently it is their impedance transformation capabilities and pulse shaping properties that are used in pulse height encoding and fast timing applications. Gain is generated in a photomultiplier by secondary emission and hence the equivalent circuit is that of an ideal current generator, as shown in **figure 1**.

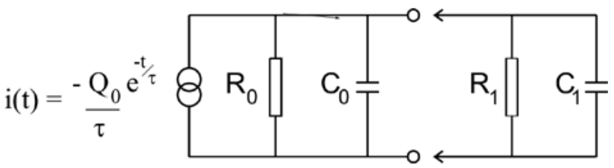


**figure 1** the equivalent circuit for a pmt is a current generator in parallel with $R_0 >> 10^9 \Omega$ and $C_0 \sim 2 - 10pF$. Most applications can be analysed in terms of an equivalent *R* and *C* combination. Without loss of generality we can take $R = R_0/R_1$ and $C = C_1 + C_0$

Circuitry capable of transforming from a current to a voltage generator is needed because most commercially available electronics is based on voltage sources and voltage amplification.

## 2 limitations on photomultiplier performance

Another consideration concerns limitations on photomultiplier performance. There is a loss in multiplier gain under continuous operation but in an unpredictable way. The major consideration is the total charge taken from the multiplier and external amplification is advisable whenever a mean anode current > 1 μA is to be drawn, as illustrated in **figure 2**. As an example, the observation of high energy gamma rays using air imaging Cerenkov

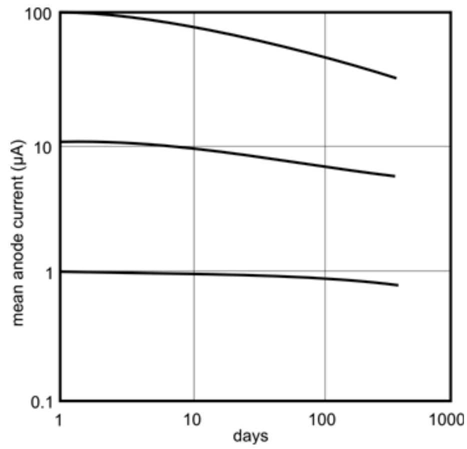


**figure 2** illustrating the loss of gain for SbCs dynodes as a function of mean anode current under conditions of constant HV and constant illumination.

telescopes is made in the presence of night sky background of intensity ~2 x $10^{12}$ photons $m^{-2}$ $sr^{-1}$ s-1. [1]. The photoelectron detection rate for a 30 mm diameter pmt is ~$10^8$ $s^{-1}$ and at $10^6$ gain this gives anode current of 16 μA. Contrary to the then current 'wisdom' Eckart Lorenz [2] rightly proposed some ten years ago that the ideal pmt for this application is one with only 6 stages operating at $10^4$ gain together with a fast, low noise, preamplifier to make up for the low gain of the photomultiplier.

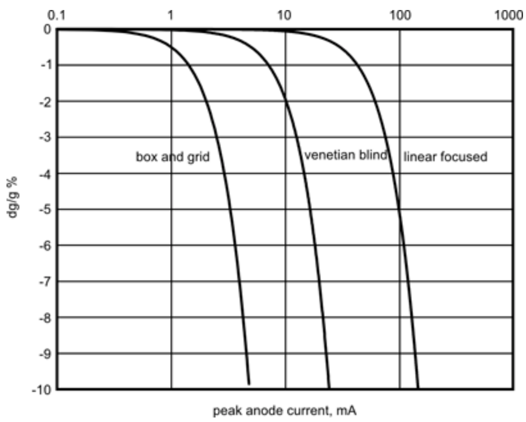


**figure 3** departure from linear gain, dg/g for three dynode types

**Figure 3** shows that pmts are also limited with regard to the peak current that can be drawn from the anode. Those with linear focused dynodes offer the best performance in signal linearity of up to about 100 mA, or equivalently 5 volts into 50 Ω. This upper limit is sufficient for most applications but where, for example, the signal is to be split several ways, then clearly additional amplification must be sought.

## 3 the photomultiplier equivalent circuit and output waveforms

In order to design and analyse electronic circuits for use with pmts we need an equivalent circuit that contains the minimal circuit elements to mimic pmt behaviour. Similarly for describing the pmt output signal, we want to use a functional representation that is simple yet adequate. The equivalent circuit in **figure 1** is sufficient for most applications, and equally valid for dc and pulsed light applications. $C_0$ is the capacitance between the anode and ground, the magnitude of which depends primarily on the area of the dynodes and on their spacing: for pmts of diameter less than 25 mm, $C_0$ *~3 pF* and for those of diameter greater than 50 mm, $C_0$ may exceed *10 pF*. The lead wires within the pmt have inductance of the order of 100 nH which together with $C_0$ constitute a tuned circuit of ringing frequency ~ 200 MHz ($\omega = (LC)^{-1/2}$). The effect of this is sometimes seen as a high frequency oscillation on the trailing edge of past pulses. Fortunately we can omit the complication of inductances in our equivalent circuit: they can be critically damped, if necessary, by the inclusion of resistors, in series with each of the last two or three dynodes in the voltage divider network.

The time signature of a single electron initiated signal, as observed at the anode of a pmt, contains all the information we need for circuit analysis purposes. Since any multiphotoelectron signal is just the superposition of a set of single photoelectron events, with a prescribed distribution in time, all we need to know is *i(t)* for single photon excitation. This enables the simulation and analysis of real scintillation and Cerenkov events, as is done in section 5. Depending on the type of multiplier, pmts have rise times ranging from 1 to about 10 ns with fall times some 2 to 3 times the rise time. To ease analysis, without departing too far from reality, we may assume a simple exponential form for i(t).

$$i(t) = (-eg/\tau).\exp(-t/\tau) \qquad \ldots(1)$$

where *e* is the electronic charge and *g* the multiplier gain and $\tau$ relates to the fall time of the single electron initiated pulse. Note that the peak current is $eg/\tau$ and that integrating *i(t)* over all *t* sums to *eg*, as it must do. The output voltage developed across *R* is

$$v_0(t) = -egR/(\tau_1 - \tau).\{\exp(-t/\tau_1) - \exp(-t/\tau)\} \quad \text{provided that } \tau \neq \tau_1 \qquad \ldots(2)$$

The choice of $\tau = \tau_1 = RC$ is not an unusual one and for this case we must take the limit $\tau \to \tau_1$ in (2) to give:

$$v_0(t) = -egtR/(\tau_1^2).\exp(-t/\tau_1) \quad \text{for } \tau = \tau_1 \qquad \ldots(3)$$

$$= -egt/(C\,\tau_1).\exp(-t/\tau_1) \qquad \ldots(4)$$

Equations (2) - (4) characterise the simplest possible amplifier that can be used with a photomultiplier – a resistor. Here the resistor performs the function of current-to-voltage conversion. Equation (2) has been plotted in **figure 4**(a) with *R = 50 Ω* for a set of *C* values, corresponding to a range of output time constants.

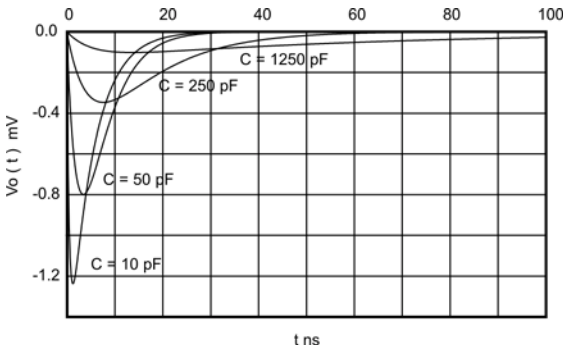


**figure 4** (a) output pulse shapes for a pmt with $\tau$ = *5ns* and anode load *R = 50 Ω*, for a range of parallel capacitors. Note how adding capacitance causes loss of pulse height.

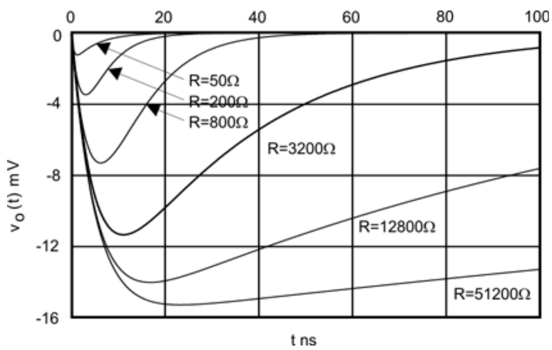


**figure 4** (b) pulse shapes for *C = 10 pF* and a range of resistor values. Here we note that the signal does not follow the input excitation when *RC* >> $\tau$.

There is a number of points to note.

- the output pulse is always negative
- the area under all pulses is the same and proportional to eg
- the output is a faithful reproduction of *i(t)* whenever $t_1 << \tau$

Equation (2) has also been plotted in **figure 4**(b) with $C = 10\ pF$ but with $R$ variable. The argument against increasing $R$ to attain the maximum pulse height is all too obvious, for we see that the output is no longer a faithful reproduction of the input. As $R$ increases, $v_0 \rightarrow eg/C$ and the signal decay times becomes progressively longer. The circuit is said to be 'integrating' the signal and although ideal for pulse height encoding, long time constants obviously restrict the rate at which events can be handled. Capacitance is the enemy of low noise amplification and we see from **figures 4**(a) and (b) that it is undesirable in other respects: it causes loss in amplitude and distortion of the input signal signature.

## 4 the case for fast amplifiers

There are two key considerations arising in the previous section. Firstly, as a current generator the pmt has limited drive capability and secondly, if pulse shapes are important (note the results of **figure 4**(b)) then the output capacitance of the pmt plus stray capacitance will place a limit on the maximum size of the load resistor that may be used – and hence the effective voltage gain. We can overcome these problems by using fast feedback amplifiers, first developed in the 1960s following the availability of fast transistors.

The feedback pair, taken from Gillespie [3] and shown in **figure 5**(a), is typical of many circuits published at that time. It has a common emitter input and hence inherently high input impedance. Gain, set by the ratio $R_f / R_s$, of 10 to 30 with a rise time of the order of 1 ns is achievable. There is scope, when mounting the amplifier within the pmt housing, to achieve extra voltage gain by increasing the size of the input resistor. However, in most instances the amplifier is remotely located and fed by coaxial cable which requires 50 Ω termination. This configuration formed the basis of the commercially available fast amplifiers of the LeCroy, NIM series (type LRS333) and in modules from other manufacturers [4]. The amplifier shown in **figure 5**(b) is an example of an early current-to-voltage or transimpedance amplifier taken from Whittaker [5]. The low impedance grounded base input configuration, with an impedance of only ~10 Ω, provides just part of the total 50 Ω input impedance. This amplifier has a gain of 10 set by $R_f / R_{in}$, low output impedance and fast rise time

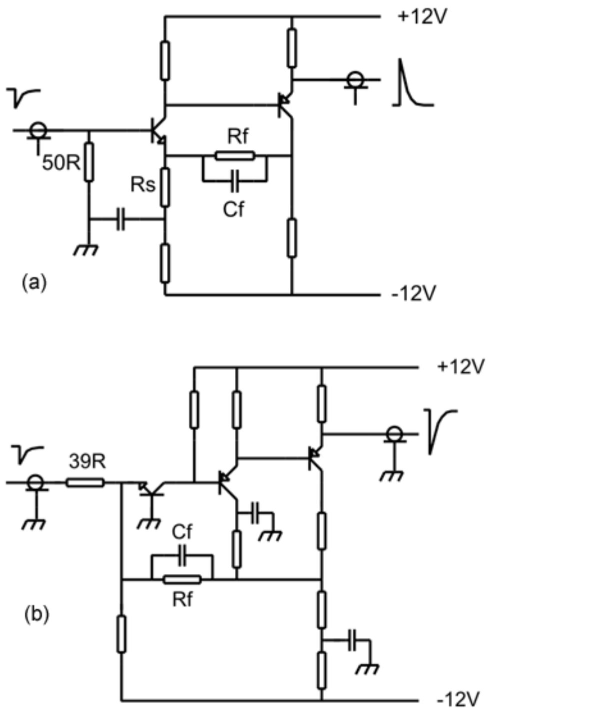


**figure 5** fast feedback amplifiers developed in the 1960s - simple, inexpensive and yet highly effective. These formed the building blocks of many commercial electronics modules of the time and they are still used today.

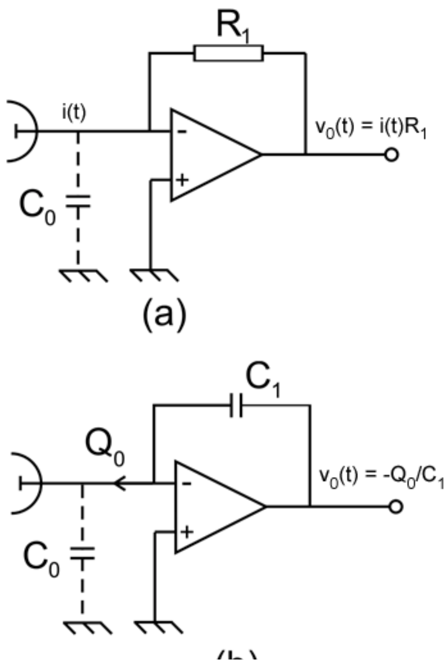


**figure 6**(a) the transimpedance Op Amp configuration that isolates the capacitance associated with the pmt output from the current resistor *R*. (b) charge sensitive transimpedance configuration which also isolates the effect of $C_0$.

Representing the transimpedance amplifier as an Op Amp configuration in **figure 6**(a) highlights the transformation of a resistor in the anode to one located in the feedback path of an Op Amp. We have thereby isolated the resistor from the unwanted influence of capacitance $C_0$ and $R_1$ now sees only a circuit board capacitance of $\sim 0.1\ pF$

and any intentionally added capacitance $C_f$ ( to provide high frequency stability or pulse shaping). Until about five years ago commercial Op Amps were unsuitable for fast pulse work or low noise applications but thanks to the mobile phone market, devices such as the CLC449 and MMIC types INA0218 became available and have been used by the MAGIC group [2] to realise fast high sensitivity transimpedance amplifiers.

## 5 the case for slow amplifiers

In contrast to handling fast pulses covered in section 4, there are applications involving scintillators where the photons generated in the process follow a decay curve with a time constant considerably longer than the single electron pulse width of the photomultiplier. NaI(Tl), for example, emits about 30 photons per keV gamma energy deposited with a time constant $\tau_s$ of 230 ns while YAP produces only 8 photons per keV but with a decay time of 30 ns. Equations **(1) – (4)** still apply but with ts in place of $\tau$. The information in scintillation events lies principally in the total charge in the pulse, the event rate and sometimes in the time of occurrence of the events.

In contrast to the treatment given in section 4, where we considered a transimpedance amplifier with feedback resistor $R_1$ as the primary feedback element, we now investigate an amplifier where the feedback element is essentially a capacitor, $C_1$, as shown in **figure 6**(b). This is known as a charge sensitive amplifier because of the way in which it functions. Straightforward circuit analysis gives

$$C_{eff} = C_0 + (1 + A)C_1 \qquad \ldots(5)$$

$$v_0 = - Q_0/C_1 \qquad \ldots(6)$$

The effective capacitance seen by the signal emanating from the pmt anode is large and $\sim AC_1$. Where A is the open-loop gain of the amplifier. This means that the signal can be transmitted down a comparatively long length of unterminated coaxial cable to such a preamplifier without loss of amplitude. Although outside the scope of this article it is worth noting the importance of **(6)** to detectors such as silicon PIN diodes and APD's and the series of papers by Goyot are worthy of study for further education [6]. The self-capacitance of APDs is relatively high compared with pmts and it changes with bias current, but equations **(5)** and **(6)** tell us that this is relatively unimportant. In practice the feedback capacitor must be discharged between every event. This may be done with an active switch connected across $C_1$ although more commonly by employing a parallel resistor and the output voltage $V_0(t)$ is given by equations **(2)** – **(4)**.

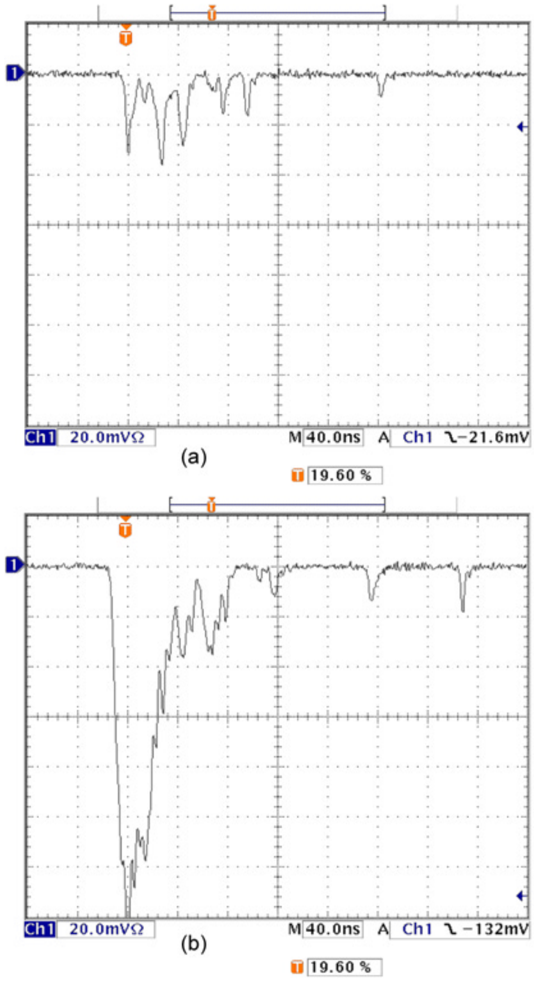


**figure 7** (a) a low energy x ray campture form $^{55}$Fe(5.9 keV) in a YAP crystal recorded on a fast oscilloscope. Note individual photoelectrons are clearly seen. (b) a 60 keV event gives a single output pulse but single electron structure is still evident, especially in the tail.

The output signal from a scintillator is statistical in nature, both with regard to the total number of photoelectrons generated and in the time interval between photoelectron detections. In both cases the statistics may be assumed to be Poisson and the interval distribution tells us about the arrival times of the individual photoelectrons.

The statistical nature of the output of fast YAP scintillator is illustrated in **figure 7**(a) where the individual photodetections generated in the absorption of a 6 keV, $^{55}$Fe x-ray are clearly delineated when measured with a 50 Ω oscilloscope. Also shown in **figure 7**(b) is an event initiated by a 60 keV, $^{241}$Am event where we observe something approaching a single pulse but with evidence of structure, particularly in the tail. These events have been simulated for a transimpedance amplifier by selecting the arrival time of single electron events to an exponential distribution and, in addition, the charge content of each single electron event is also subject to the statistical pulse height distribution that describes the single electron response of the pmt used. The contribution made to the output from each photo-electron over time is determined and summed to give $v_0(t)$ as shown in **figure 8**.

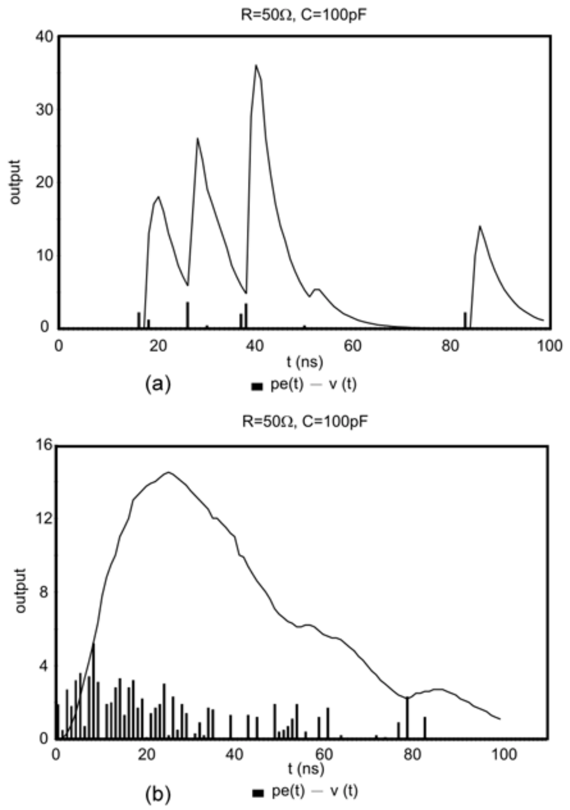


**figure 8** (a) Monte Carlo simulation event, such as one from $^{55}$Fe, in a YAP crystal producing only 8 photoelectrons. (b) simulation of a $^{241}$Am event of 60 keV where the total number of photoelectrons is 100 and less fine structure is evident.

We note that the time constant selected here is insufficient to give any semblance of a single pulse in **(a)** but is sufficient in the 100 photoelectron event in **(b)** to give a pulse the height which, to a first approximation , is proportional to the total number of photoelectrons generated. Note in **figure 8**(b) that single photoelectrons, shown as vertical bars, are arriving even after 80 ns from the initiation of the event – this tells us something about the ability or otherwise of this amplifier configuration to resolve scintillations at high count rates.

## 6 amplifiers for energy and timing measurements – *RC-CR* pulse shaping

There are situations where accurate assignment of the time of occurrence of an event is of prime concern. Consider the waveforms of **figure 9**(a) applied to a threshold discriminator set to 1.0 V. We see that the smallest pulse crosses the threshold after 0.5 μs whereas a pulse of four times the amplitude crosses after only 100 ns. This walk as it is called can be reduced by following *A1* with a *CR* configuration, A2, as shown in **figure 9**(b) and $v_0(t)$ is given by:

$$v_0(t) = -Q_0\tau_1^2\tau_2/(C_1(\tau_1 - \tau_s)(\tau_1 - \tau_2))\{1/\tau_1 \exp(-t/\tau_1) - 1/\tau_2 \exp(-t/\tau_2)\}$$
$$+Q_0\tau_1\tau_2\tau_s/(C_1(\tau_1 - \tau_s)(\tau_s - \tau_2))\{1/\tau_s \exp(-t/\tau_s) - 1/\tau_2 \exp(-t/\tau_2)\} \quad \ldots(7)$$

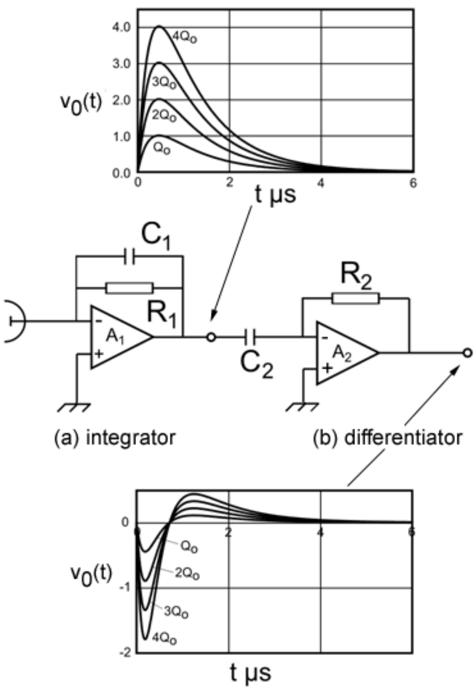


**figure 9** (a) unipolar output waveforms for NaI(Tl) scintillations from a pmt with gain of $10^6$ initiated by a 1000 to 4000 photo-electron signals. The walk may be reduced by 'differentiating' these signals, as in (b).

The waveform of (7) is bipolar and crosses the zero axis at the same time, relative to the initiation, regardless of amplitude. A discriminator set at + 50 mV, or preferably even lower, will provide improved timing fidelity - such circuits are called zero-crossing discriminators. Commercial shaping amplifiers are more complex in their operation than described above but the principles outlined still apply. The industry standard discriminator for timing is the constant fraction type which is based on the considerations outlined above. A further advantage derives from using bipolar pulses: reduced baseline shift with rate. This is otherwise a serious problem in ac-coupled systems.

## 7 conclusions

A photomultiplier detection system will invariably have several elements: such as preamplifier, main or shaping amplifiers followed by a multichannel analyser or discriminator. The challenge lies in how to apportion gain or sensitivity amongst these various sub-systems. The guiding principle must always be to ensure that each sub-system is operating within its range of ideal performance and to acknowledge the boundaries of gain capability, linearity and fatigue that apply to photomultipliers. Using the appropriate 'integrating' amplifier can mitigate the statistical nature of the pmt output - inescapable in low energy scintillation spectroscopy. Timing, independent of pulse amplitude, can be achieved by employing an amplifier that produces a bipolar output. Photomultipliers are current generators and the transimpedance amplifier, with its near zero input impedance, provides the ideal means to make the required current-to-voltage conversion for signal handling purposes.

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